Journal of Statistical Physics, Vol. 71, Nos. 5/6, 1993

System–Reservoir Interaction with Stochastic Coupling Parameters

M. Mañas,[1] J. M. R. Parrondo,[2] and F. J. de la Rubia[3]

Received April 7, 1992; final January 5, 1993

We consider the problem of a system coupled to an ensemble of independent harmonic oscillators acting as a reservoir. We use an extension of the functional derivative technique to analyze some of the effects of adding stochastic terms to the system–reservoir coupling parameters. Two approaches (quantum master equation and Langevin equation) are considered and their ranges of validity and differences are examined.

KEY WORDS: System–reservoir interaction; stochastic coupling; quantum master equation; Langevin equation.

1. INTRODUCTION

The Hamiltonian

$$H_0 = H_S + H_B + \varepsilon H_I \tag{1}$$

with ($\hbar = 1$)

$$\begin{aligned} H_S &= \omega_0 a^\dagger a \\ H_B &= \sum_{i=1}^{N} \omega_i A_i^\dagger A_i \\ H_I &= \sum_{i=1}^{N} [g_i A_i^\dagger a + g_i^* A_i a^\dagger] \end{aligned} \tag{2}$$

[1] Departamento de Física Teórica, Universidad Complutense, 28040 Madrid, Spain.
[2] Departamento de Física Aplicada I, Universidad Complutense, 28040 Madrid, Spain.
[3] Departamento de Física Fundamental, Universidad a Distancia, Apdo. de Correos 60141, 28080 Madrid, Spain. Author to whom all correspondence should be addressed.

0022-4715/93/0600-1157$07.00/0

has been the subject of much theoretical work[1–7] and widely considered in the literature to describe different physical situations: interaction between phonon modes produced by anharmonicity of the lattice,[8] damped vibration of a diatomic molecule,[9] spin relaxation in nuclear magnetic resonance,[10] or, in quantum optics, to describe either the interaction of a two-level system and radiation or the relaxation of one mode of a field when it interacts with the other modes.[11–13]

Our purpose in this work is to extend some of the results regarding the Hamiltonian (1) when a time-dependent stochastic term $\xi_i(t)$ is added to the coupling constants g_i. The resulting Hamiltonian is then given by

$$H = H_0 + \varepsilon \sum_{i=1}^{N} \{\xi_i(t)\, A_i^\dagger a + \xi_i^*(t)\, A_i a^\dagger\} \tag{3}$$

where $\xi_i(t)$ are complex colored Gaussian noises with zero mean value and correlation functions

$$\begin{aligned} \langle \xi_i(t)\, \xi_j(t') \rangle &= \gamma_{ij}(t-t') \\ \langle \xi_i(t)\, \xi_j^*(t') \rangle &= \Gamma_{ij}(t-t') \end{aligned} \tag{4}$$

We try to model *external* fluctuations acting on the whole system. An important example of such fluctuations in the coupling coefficients g_i occurs when the Hamiltonian (1) models, in the two-level approximation, the interaction between an atom or molecule and an electromagnetic field. In this case, the original Hamiltonian of the electron interacting with the field is

$$H_{\text{at}} + H_{\text{rad}} + \frac{e}{m} \mathbf{r} \cdot \mathbf{E} \tag{5}$$

When only the transitions between two levels of the spectrum of H_{at} are relevant one can project the operator (5) onto the subspace spanned by the eigenfunctions $\psi_a(\mathbf{r})$ and $\psi_b(\mathbf{r})$ corresponding to these levels.[13] When considering atoms or molecules without intrinsic polarization, the Hamiltonian (1) is recovered if one expresses the field $\mathbf{E}$ in terms of its normal modes $A_i^\dagger$, A_i and assumes the *rotating wave approximation* (RWA).[3,4,11,12] Then the coefficients g_i are proportional to $\not{p} \cdot \boldsymbol{\varepsilon}_i$, where $\boldsymbol{\varepsilon}_i$ is the polarization vector of the ith mode, and

$$\not{p} = e \int d\mathbf{r}\, \psi_a(\mathbf{r})\, \mathbf{r} \psi_b^*(\mathbf{r}) \tag{6}$$

However, the wave functions $\psi_a(\mathbf{r})$ and $\psi_b(\mathbf{r})$ are deformed by the forces that act on the atom when it is out of its equilibrium position due to

thermal vibrations. Thus, the mechanism of this deformation is a random Stark effect and it results in fluctuations in the frequency ω_0 between the levels and also in the matrix element $\not{p}$. These fluctuations have been previously modeled as a white noise in order to explain the difference between the decay times of the level populations and the off-diagonal elements of the density matrix,[13] but the effects of the fluctuations in the coupling constants g_j are still unexplored.

A complete study of this problem should be carried out considering simultaneously the random shift in the frequency ω_0, the intrinsic dipole of each level that results in cubic terms in the Hamiltonian, feedback terms due to the influence of the motion of the electron on the nucleus, and the fully coupled Hamiltonian without making the RWA. However, it seems likely that the novelties appearing with the modification (3) in (1) will essentially remain in the complete treatment. Our aim in this paper is to stress the theoretical interest of the problem and to discuss the use of different techniques in order to solve it.

On the other hand, one could argue that this situation can be modeled with two reservoirs: the radiation and the vibration field of the lattice. This will lead to transitions due to absorption and emission of either photons or phonons, but not to the main effect which is modeled by the noise, i.e., the fluctuations in the polarizability, since this effect can be taken into account only using the electronic wave functions. In this sense our approach goes beyond the two-level approximation.

Another possible interesting application is considered in refs. 5 and 7, where it is proved that a system tunneling between two wells in the presence of a dissipative environment can be described by a two-level system linearly coupled to an oscillator ensemble, and such that the coupling coefficient is proportional to the distance between the wells of the potential. Then it is easy to conceive situations in which this distance fluctuates and these fluctuations could again be modeled by adding stochastic terms to the coupling parameters. However, in this situation the difference between the full Hamiltonian and its RWA version is important[5,14] and the former should be used in the analysis.

There are some precedents for our model that are worth mentioning. Mencia Bravo *et al.*[15] dealt with a related problem: a classical system in a heat bath with an additive external noise. In the quantum case, Faid and Fox[16] proposed a stochastic coupling between a system and a heat bath as a phenomenological mechanism of relaxation for the bath. However, they set the statistical properties of the coupling terms as a whole, preventing nonconservative transitions. We here consider the noises and the bath as independent objects, and obtain as a result nonconservative transitions.

Two techniques are presented in this paper. The first is the quantum

master equation (QME).[6,7] This is an equation for the density matrix of the system averaged over the bath variables, and is used in order to calculate the transition probabilities between energy levels of the system. These are the coefficients of the master equation for the level population, i.e., the diagonal part of the density matrix. In this approach, carried out in Section 2, one can obtain the stationary level populations, and the deviations from the deterministic case (thermalization) are discussed. The second one, the Langevin approach, is considered in Section 3, presenting equations for the first moment of a and $a^\dagger$ that go beyond the approximations yielding the QME. The possibility of going beyond the first moment is also examined.

The stochastic terms are treated in Sections 2 and 3 using what we call the quantum best Fokker–Plank equation (QBFPE).[17] This equation is obtained by averaging the quantum evolution equations over the realizations of the noises, and may be found either using cummulant techniques[4] or in the context of the best Fokker–Plank approximation for stochastic equations perturbed by colored noise.[18]

2. QUANTUM MASTER EQUATION

Our purpose in this section is to obtain an evolution equation for the density matrix averaged over the noises, and by a standard projection technique[6,7] to eliminate the bath variables. The derivation of such an equation can be carried out by using cumulant expansions[4] or by a suitable extension of functional derivative techniques related to the "best Fokker–Planck approximation"[17] for colored-noise-perturbed systems. Following this latter technique, we start with a general quantum system with stochastic Hamiltonian

$$H(t) = H_0 + \sum_k \xi_k(t)\, H_k \tag{7}$$

where H_i are self-adjoint operators and $\xi_k(t)$ real Gaussian noises with zero mean and correlation given by

$$\langle \xi_k(t)\, \xi_{k'}(t') \rangle = \Gamma_{kk'}(t-t') \tag{8}$$

The evolution equation for the operators A_t in the Heisenberg picture, after averaging over the noises, can be found to be

$$\langle \dot{A}_t \rangle = i\langle [H_0, A]_t \rangle - \sum_k \langle [D_k(t), [H_k, A]]_t \rangle \tag{9}$$

where the subscript t denotes the (stochastic) evolution in the Heisenberg picture and $\langle \cdot \rangle$ is the average over the noises. The operators $D_k(t)$ depend on $H(t)$ in such a way that unless the noises are delta correlated, these are stochastic operators.[17] The approximation which yields the so-called best Fokker–Planck equation (BFPE) in the classical case replaces the exact $D_k(t)$ by deterministic expressions. This approximation is valid for weak noises and short correlation times and its range of validity has been subject of a number of studies.[19] In the quantum case the approximated $D_k(t)$ reads

$$D_k(t) = \sum_{k'} \int_0^t ds\, \Gamma_{kk'}(s)\, e^{-isH_0} H_{k'} e^{isH_0} \tag{10}$$

With this approximation it is possible to find a density matrix ρ_t such that it represents the actual state of the system in the sense that, if ρ_0 is the initial state, then

$$\mathrm{Tr}[\rho_t A] = \mathrm{Tr}[\rho_0 \langle A_t \rangle] \tag{11}$$

This state reproduces all the mean values for a given time, since A is an arbitrary observable. The only information which is lost with this treatment is that of time correlations, i.e., mean values of pairs of observables measured at two different times. Deriving (11) and introducing (9) in it, we finally obtain the evolution equation for ρ_t, referred to as the quantum BFPE (QBFPE),

$$\dot{\rho}_t = -i[H_0, \rho_t] - \sum_k [H_k, [D_k(t), \rho_t]] \tag{12}$$

with $D_k(t)$ given by (10).

We will apply the above formalism to the Hamiltonian (1) with the interaction term given by (3). Although the noises in (3) are complex, one can reduce this problem to one with real noises by just splitting $\xi_i(t)$ into real and imaginary parts.

Once set the Hamiltonian, it is straightforward to obtain the QBFPE for the density matrix ρ_t, which reads

$$\dot{\rho}_t = -i[H_0, \rho_t] - \varepsilon^2 \sum_{i=1}^{N} \{[V_i, [D_i(t), \rho_t]] + [V_i^\dagger, [D_i(t)^\dagger, \rho_t]]\} \tag{13}$$

where $V_i = A_i^\dagger a$ and

$$D_i(t) = \sum_{j=1}^{N} \int_0^t ds\, e^{-iH_0 s} [\gamma_{ij}(s)\, V_j + \Gamma_{ij}(s)\, V_j^\dagger]\, e^{iH_0 s} \tag{14}$$

The parameter ε is considered to be small, in such a way that the motion induced by the coupling is much slower than the motion of the isolated system or the bath. Thus, we can perform an expansion on ε which is equivalent to an adiabatic elimination of the bath variables in the interaction picture.[6,7] As usual, we average over the bath variables in order to get an equation for the reduced density matrix. We define the projectors

$$P\rho = \rho(T) \otimes \mathrm{Tr}_B \rho, \qquad Q = \mathbf{1} - P \tag{15}$$

where Tr_B is the trace over the bath variables, $\rho(T)$ is the Gibbs equilibrium state of the bath at a temperature T, and

$$v_t = P\rho_t, \qquad w_t = Q\rho_t \tag{16}$$

Applying these projectors to (13), we obtain the evolution equations for v_t and w_t. If one assumes the bath and the system to be uncorrelated and the former in thermal equilibrium at the initial time, i.e., $\rho_0 = \rho(T) \otimes \hat{\rho}_0$, then it is found, after eliminating w_t, that v_t satisfies, up to second order in ε, the evolution equation

$$\dot{v}_t = -i[H_S, v_t] - 2\varepsilon^2 \,\mathrm{Re}\, P \sum_{i=1}^{N} \int_0^t ds(|g_i|^2 + \Gamma_{ii}(s))\, e^{i(\omega_i - \omega_0)s}[V_i, [V_i^\dagger, v_t]] \tag{17}$$

where we have used the notation $\mathrm{Re}\, A = \frac{1}{2}(A + A^\dagger)$. Since we have assumed a thermal bath with noninteracting oscillators, the correlation among the different noises, $\Gamma_{ij}(t)$ with $i \neq j$, does not affect the final result. Hereafter we suppose the noises to be uncorrelated,

$$\Gamma_{ij}(s) = \delta_{ij}\Gamma_i(s) \tag{18}$$

Finally, if $\mathrm{Tr}_B(\rho(T)\, A_i^\dagger A_i) = n_i(T)$ is the mean number of quanta with energy ω_i, we can find for $\hat{\rho}_t = \mathrm{Tr}_B \rho_t$ the quantum master equation (QME)

$$\begin{aligned}\dot{\hat{\rho}} = &-i(\omega_0 + \Delta\omega_0)[a^\dagger a, \hat{\rho}_t] + \tfrac{1}{2}K(\bar{N}+1)[a\hat{\rho}_t a^\dagger - a^\dagger a\hat{\rho}_t - \hat{\rho}_t a^\dagger a] \\ &+ \tfrac{1}{2}K\bar{N}[2a^\dagger\hat{\rho}_t a - aa^\dagger\hat{\rho}_t - \hat{\rho}_t aa^\dagger]\end{aligned} \tag{19}$$

where

$$\begin{aligned}\Delta\omega_0 &= \varepsilon^2 \,\mathrm{Im} \sum_{i=1}^{N} \int_0^\infty ds(|g_i|^2 + \Gamma_i(s))\, e^{i(\omega_i - \omega_0)s} \\ K &= 2\varepsilon^2 \,\mathrm{Re} \sum_{i=1}^{N} \int_0^\infty ds(|g_i|^2 + \Gamma_i(s))\, e^{i(\omega_i - \omega_0)s} \\ K\bar{N} &= 2\varepsilon^2 \,\mathrm{Re} \sum_{i=1}^{N} n_i(T) \int_0^\infty ds(|g_i|^2 + \Gamma_i(s))\, e^{i(\omega_i - \omega_0)s}\end{aligned} \tag{20}$$

In the latter expressions we have made $t \to \infty$ in the upper limits of the integrals, which is the result for adiabatic elimination. The replacement is justified in the noise terms assuming t large enough to lie out of the support of $\Gamma(s)$. Thus, the form of the QME (19) is the same as those derived in the absence of noises but with different expressions for the frequency shift $\Delta\omega_0$ and the factors K and $\bar{N}$. These differences may have important physical consequences. In our case we will consider the process of emission and absorption of a photon or phonon. From (19) we can calculate the transition rates, which are the coefficients of the master equation for the level populations $P_n = \langle n | \hat{\rho}_t | n \rangle$, and are obtained taking the expectation value of (19) on the eigenvectors of H_S:

$$\begin{aligned} w_{\text{em}}(n) &= nK(\bar{N}+1) \\ w_{ab}(n) &= (n+1)\, K\bar{N} \end{aligned} \tag{21}$$

If $[|g_j|^2 + \Gamma_{jj}(s)]$ does not depend on s, the dominant transitions are those that conserve the energy. The standard treatment is to consider large times and a continuous set of possible final states, which can be thought of as a thermodynamic limit in the bath variables. Then one obtains the Fermi golden rule, which gives constant transition rates. Formally, the Fermi golden rule can be derived from (20) and (21) by taking $t \to \infty$ and replacing the sum by an integral over ω with a state density $D(\omega)$, i.e.,

$$\begin{aligned} w_{\text{em}} &= 2\varepsilon^2 n_S \,\text{Re} \int_0^\infty d\omega\, D(\omega)[n(\omega)+1] \\ &\quad \times \int_0^\infty ds\, e^{i(\omega-\omega_0)s}[|g(\omega)|^2 + \Gamma(\omega, s)] \\ w_{\text{ab}} &= 2\varepsilon^2 (n_S+1)\,\text{Re} \int_0^\infty d\omega\, D(\omega)\, n(\omega) \\ &\quad \times \int_0^\infty ds\, e^{i(\omega-\omega_0)s}[|g(\omega)|^2 + \Gamma(\omega, s)] \end{aligned} \tag{22}$$

Let us split the deterministic and stochastic contributions to the rates,

$$\begin{aligned} w_{\text{em}} &= w_{\text{em}}^{(\text{det})} + \Delta w_{\text{em}} \\ w_{\text{ab}} &= w_{\text{ab}}^{(\text{det})} + \Delta w_{\text{ab}} \end{aligned} \tag{23}$$

The deterministic part is given by the Fermi golden rule,

$$\begin{aligned} w_{\text{em}}^{(\text{det})} &= 2\pi\varepsilon^2 n_S D(\omega_0)[n(\omega_0)+1]\,|g(\omega_0)|^2 \\ w_{\text{ab}}^{(\text{det})} &= 2\pi\varepsilon^2 (n_S+1)\, D(\omega_0)\, n\,(\omega_0)\,|g(\omega_0)|^2 \end{aligned} \tag{24}$$

while the stochastic part is

$$\Delta w_{\rm em} = \varepsilon^2 n_S \int_{-\infty}^{\infty} d\omega\, D(\omega)[n(\omega)+1]\,\tilde{\Gamma}(\omega, \omega_0-\omega)$$
$$\Delta w_{\rm ab} = \varepsilon^2 (n_S+1) \int_{-\infty}^{\infty} d\omega\, D(\omega)\, n(\omega)\,\tilde{\Gamma}(\omega, \omega_0-\omega) \tag{25}$$

where $\tilde{\Gamma}(\omega, \Omega)$ is the Fourier transform of the correlation function

$$\tilde{\Gamma}(\omega, \Omega) = \int_{-\infty}^{\infty} ds\, e^{-i\Omega s}\Gamma(\omega, s)$$

and we have applied that $\Gamma_{ij}^*(t) = \Gamma_{ji}(-t)$, which is clear from the definition (4).

One can see that only the number of excitations and coupling constants of the resonant mode are involved in the deterministic contribution. On the contrary, the stochastic part can introduce a dependence on the other modes. For example, if the noise spectra are peaked around a frequency $\Omega_{\rm no}$, then the modes with frequencies close to $\omega_0 - \Omega_{\rm no}$ will contribute to the transition rates. This is a remarkable effect of the external fluctuations, which permit absorption and emission of excitations corresponding to non-resonant modes. If the correlations are real, then the Fourier transform is an even function in Ω and the absorption spectrum will have one peak in ω_0 due to the deterministic part and two smaller peaks in $\omega_0 \pm \Omega_{\rm no}$ due to the stochastic term $\Delta w_{\rm em}$.

To end this section we stress that the QME also gives the frequency shift, i.e., the increment on the free frequency ω_0 due to the presence of the bath. The stochastic forces produce new terms in the frequency shift. This stochastic correction $\Delta\omega_0^{(\rm no)}$ can be written in terms of the Fourier transform of $\Gamma(\omega, t)$, taking the limit of a continuous set of modes,

$$\Delta\omega_0^{(\rm no)} = \varepsilon^2 \int_0^{\infty} d\omega\, D(\omega)\, {\rm PV} \int_{-\infty}^{\infty} d\Omega\, \frac{\tilde{\Gamma}(\omega, \Omega)}{\Omega + \omega - \omega_0} \tag{26}$$

where PV means the principal value of the integral over Ω.

3. LANGEVIN APPROACH

The starting point of the Langevin approach[3,4,7,11–13] are the Heisenberg equations for the operators a, $a^\dagger$, A_i, and $A_i^\dagger$ and it consists in

the elimination of the bath variables A_i, $A_i^\dagger$ yielding a closed equation for a and $a^\dagger$, which reads, for the deterministic Hamiltonian (1) with (2),

$$\dot{a} = -i\omega_0 a - \int_0^t ds\, \phi(t-s)\, a(s) + F(t) \tag{27}$$

with the dissipation kernel

$$\phi(t) = \sum_{k=1}^{N} |g_k|^2 e^{-i\omega_k t} \tag{28}$$

and the Langevin force, which is an operator,

$$F(t) = i \sum_{k=1}^{N} g_k A_k e^{-i\omega_k t} \tag{29}$$

If one tries to generalize this procedure to the stochastic case, one finds a stochastic dissipation kernel. Then we have, acting on the system observables, two sources of fluctuations, one due to the bath, which is also present in the deterministic case, and the other due to the external noises. The resulting Langevin equation contains terms with the square of a Gaussian noise, and then it is no longer possible to apply the simple standard theory of quantum stochastic equations. To keep the calculations simple enough, in this work we choose to generalize as much as possible the results of the quantum Langevin equation to the stochastic case, and not to try the methods existing in the literature to treat nonlinear Gaussian noises.[20]

Using (9), we can write exact equations for the average of any observable of the whole system over all the realizations of the noises. We call this average $\langle A_t \rangle_{\text{no}}$. One can reproduce the derivation of the quantum Langevin equation, eliminating the bath variables and obtaining stochastic equations for $\langle a_t \rangle_{\text{no}}$ and $\langle a_t^\dagger \rangle_{\text{no}}$. The stochasticity in these equations comes from the presence of the bath. Then one can use these equations to calculate the different moments of the stochastic operators $\langle a_t \rangle_{\text{no}}$ and $\langle a_t^\dagger \rangle_{\text{no}}$, i.e., the averages $\langle\langle a_t^\dagger \rangle_{\text{no}}^n \langle a_t \rangle_{\text{no}}^m \rangle$, where $\langle \cdot \rangle$ means the average over the initial condition of the bath. But these are not useful quantities. For example, the mean value of the energy will be given by $\langle\langle a_t^\dagger a_t \rangle_{\text{no}} \rangle$ and the commutation relation which is preserved under the evolution is $\langle\langle [a_t, a_t^\dagger] \rangle_{\text{no}} \rangle = \mathbf{1}$.

Thus, the meaningful averages are only those carried out over the initial conditions of the bath and over the realizations of the noises simultaneously. We will derive the Langevin equation for $\langle a_t \rangle_{\text{no}}$, but this will be useful only to calculate the mean value of a_t and then it will give

information only about the dissipation, but the stochastic force appearing in the equation will not have any physical meaning. Then we are forced to repeat the calculation for $\langle a_t^\dagger a_t \rangle_{\text{no}}$, obtaining the equation for the mean value of the energy, but this can be done in closed form only under the strong assumption that the interaction term is purely random, i.e., that $g_k = 0$, and will not be considered here.

In the equation for the mean value of a_t we can use the exact version (9), i.e., an equation containing a memory term due to the finite correlation time of the noises. This equation reads, for the same Hamiltonian of (9),

$$\langle \dot{A}_t \rangle = i\langle [H_0, A]_t \rangle - \sum_k \langle [G_k(t), [H_k, A]_t] \rangle \tag{30}$$

where

$$G_k(t) = \sum_j \int_0^t ds\, \Gamma_{jk}(t-s)\, H_j(s) \tag{31}$$

The BFPA carried out in Section 3 replaces $H_j(s)$ by the free evolution, the evolution without any interaction term. Here it is possible to preserve some of the memory effects on the system variables, but not in the bath variables. We will make use of the following approximations for times in which the integrand of (31) is appreciable, i.e., for $|t-s| \lesssim \tau_c$, τ_c being the maximum of the correlation times of the noises:

$$V_j(s) \simeq e^{-i\omega_j(t-s)} A_j^\dagger(t)\, a(s) \tag{32}$$

and

$$\begin{aligned} [a(s), A_j^\dagger(t)] &= 0 \\ [a(s), a(t)] &\simeq 0 \\ [a(s), a^\dagger(t)] &\simeq e^{i\omega_0(t-s)}\mathbf{1} \end{aligned}$$

With these approximations and assuming $\Gamma_{jk}(t) = \delta_{jk}\Gamma_k(t)$, one has, dropping out the brackets and the subscripts,

$$\begin{aligned} \dot{a}(t) = &-i\omega_0 a(t) - i\sum_k g_k^* A_k(t) - \sum_k \int_0^t ds\, \Gamma_k(t-s) \\ &\times e^{-i\omega_k(t-s)} a(s) \end{aligned} \tag{33}$$

$$\dot{A}_k(t) = -i\omega_k A_k(t) - i g_k a(t) - \Omega_k(t)\, A_k(t) \tag{34}$$

where

$$\Omega_k(t) = \int_0^t ds\, \Gamma_k(s)\, e^{-i(\omega_0 - \omega_k)s} \tag{35}$$

Eliminating the bath variables $A_k(t)$, one has finally

$$\dot{a}(t) = -i\omega_0 a(t) - \int_0^t ds\, \Phi(t, s)\, a(s) + \mathscr{F}(t) \tag{36}$$

with the dissipation kernel

$$\Phi(t, s) = \sum_k \{\exp[-i\omega_k(t-s)]\} \left\{ \Gamma_k(t-s) + |g_k|^2 \times \exp\left[-\int_s^t \Omega_k(t')\, dt' \right] \right\} \tag{37}$$

and the fluctuating force

$$\mathscr{F}(t) = -i \sum_k g_k^* A_k(0) \exp(-i\omega_k t) \exp\left[-\int_0^t \Omega_k(t')\, dt' \right] \tag{38}$$

For large enough times $t, s \gg \tau_c$ the dissipation kernel becomes stationary, i.e., depending only on the difference $\Phi(t, s) = \Phi(t-s)$, where

$$\Phi(t) = \sum_k e^{-i\omega_k t} [\Gamma_k(t) + |g_k|^2 e^{-t\Omega_k}] \tag{39}$$

with

$$\Omega_k = \lim_{t \to \infty} \Omega_k(t)$$

Finally, taking the average over the initial condition of the bath, one has

$$\langle \dot{a}(t) \rangle = -i\omega_0 \langle a(t) \rangle - \int_0^t ds\, \Phi(t-s) \langle a(s) \rangle \tag{40}$$

It is interesting to note that the QME yields an equation for the first moment which is obtained from (40) retaining only the second-order terms in g_k and $\Gamma_k(t)$. This is equivalent to making in (40) the approximations

$$\Omega_k \simeq 0$$

$$\langle a(s) \rangle \simeq e^{i\omega_0(t-s)} \langle a(t) \rangle$$

In (40) one can see that the deterministic coupling, i.e., the coefficients g_k, enters in the equation with the exponential of $-t\Omega_k$. Then, this effect is only important in a time scale given by Ω_k^{-1}, which is very large for weak noises. What is an interesting novelty is that the time scale of the dissipation kernel can be reduced by the presence of the noises if these are strong enough. In that case, the relevant time is no longer the inverse of the bandwidth, but the correlation time of the noises τ_c and the time Ω_k^{-1}. This effect is lost in the QME, where the deterministic and stochastic terms appeared in the same way. The exponential factor that now arises in the deterministic term comes from the effect of the noise on the evolution of the bath variables. Without noises these variables are affected by the motion of the system and the dissipation is given by the feedback of this action. In the presence of strong noises the interaction between the system and the bath is random and a dissipation due to this randomness is created both on the system and on the bath, and this causes the vanishing of the effect of the bath on the system.

4. CONCLUSIONS

A comprehensive study of the problem of stochastic interaction between a one-degree-of-freedom system (two-level system or harmonic oscillator) and a Bose field has been presented. Two standard approaches for the deterministic problem have been extended in order to include the stochastic effects. In that case nonconservative transitions yielding non-thermal stationary states are allowed. In the simpler white noise case the transitions can be done by means of absorption or emission of a photon or phonon of any frequency, while if the noises are colored, there are peaks around ω_0 given by the spectra of the noises. There is also an extra shift in the frequency ω_0 that vanishes in the white noise case. These results are obtained using the QME.

The extension of the Langevin approach has been carried out only for the first moment of the relevant variables. Nevertheless it has provided an important effect not captured with the QME, since it is possible to obtain that the deterministic dissipation kernel is decreased by the presence of the noises.

ACKNOWLEDGMENTS

Two of us (J.M.R.P. and F.J.R.) acknowledge financial support from the Dirección General de Investigación Científica y Técnica (DGICYT) projects PB88-0140 and PB91-0222.

REFERENCES

1. G. W. Ford, M. Kac, and P. Mazur, *J. Math. Phys.* **6**:504 (1965).
2. P. Ullersma, *Physica* **32**:27 (1966).
3. K. Lindenberg and B. J. West, *Phys. Rev. A* **30**:568 (1984), and references therein.
4. K. Lindenberg and B. J. West, *The Nonequilibrium Statistical Mechanics of Open and Closed Systems* (VCH, New York, 1990).
5. A. J. Legget, S. Chakravarty, A. T. Dorsey, M. P. A. Fisher, A. Garg, and W. Zwerger, *Rev. Mod. Phys.* **59**:1 (1987).
6. C. W. Gardiner, *Handbook of Stochastic Methods* (Springer, Berlin, 1985).
7. C. W. Gardiner, *Quantum Noise* (Springer, Berlin, 1991).
8. A. H. Opie, *Phys. Rev.* **172**:640 (1968).
9. A. Nitzan and J. Jortner, *Mol. Phys.* **25**:713 (1973); A. Nitzan, S. Mukamel, and J. Jortner, *J. Chem. Phys.* **60**:3929 (1974).
10. A. Redfield, *Adv. Mag. Resonance* **1**:1 (1965).
11. H. Haken, Laser theory, In *Encyclopedia of Physics*, Vol. xxv/2a (Springer, Berlin, 1970).
12. W. H. Louisell, *Quantum Statistical Properties of Radiation* (Wiley, New York, 1973).
13. M. Sargent III, M. O. Scully, and W. E. Lamb, Jr., *Laser Physics* (Addison-Wesley, Reading, Massachusetts, 1974); P. Meystre and M. Sargent, *Elements of Quantum Optics* (Springer, Berlin, 1990).
14. P. Pfeifer, *Phys. Rev. A* **26**:701 (1982).
15. J. Mencia Bravo, R. M. Velasco, and J. M. Sancho, *J. Math. Phys.* **30**:2023 (1989).
16. K. Faid and R. F. Fox, *Phys. Rev. A* **35**:2684 (1987).
17. J. M. R. Parrondo, M. Mañas, and F. J. de la Rubia, *J. Phys. A* **23**:2363 (1990).
18. J. M. Sancho, M. San Miguel, S. L. Katz, and J. D. Gunton, *Phys. Rev. A* **27**:1589 (1982); K. Lindenberg and B. J. West, *Physica A* **128**:25 (1984).
19. P. Jung and P. Hänggi, *Phys. Rev. Lett.* **61**:11 (1988); L. Ramírez-Piscina, J. M. Sancho, F. J. de la Rubia, K. Lindenberg, and G. P. Tsironis, *Phys. Rev. A* **40**:2120 (1989); M. M. Klosek-Dygas, B. J. Matkowsky, and Z. Schuss, *Phys. Rev. A* **38**:2605 (1988).
20. M. San Miguel and J. M. Sancho, *Z. Phys. B* **43**:361 (1981).